DETERMINATION OF RANDOM MOVEMENT CHARACTERISTIC PARAMETERS OF MECHANICAL MODELS WITH FINITE NUMBER OF DEGREES OF FREEDOM (ISOTHERMAL MODE)

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Abstract. *In this paper, it is considered linear elastic vibration rod (bar Bernoulli-Euler) with concentrated mass M discrete points of a mechanism R (RRR) with random excitation by the unknown. The mathematical models achieved vary over time, with weightings given random function of time and thus directly obtain the statistical parameters of the dynamic response.*
Keywords: *vibration, random, mechanism, model, response*

1. INTRODUCTION

As demonstrated in the example treated in [5], the actual calculation of the static characteristics of the dynamic response – when the mathematical model is a system of partial differential equations - it is difficult, especially when it is considered decoupling of the equations of the model and the condition rigid solid motion is complex.

In these situations, to solve the mathematical model vary over time, we can use an iterative algorithm – successive approximations - as follows:

- We removed from the structure of the mathematical model those terms that give time-varying quality. Obtain a time invariant mathematical model that can be solved for most conditions, the limit of plane mechanisms and spatial structure, using Laplace and finite Fourier transforms one of sine and cosine.
This gives an algebraic transformation of the mathematical model, which leads to full inversion transforms, a solution $\overline{u^{(0)}}$.

It is sometimes necessary to remove from the structure of the mathematical model terms of coupling in order to use the finite Fourier transforms with respect to the abscissa axis of the bar.
We decide $T_1(\{u\}^{(0)},t)$, $T_2(\{u\}^{(0)},t)$, $T_3(\{u\}^{(0)},t)$ are removed terms.
- We calculate $T_1(\{u\}^{(0)},t)$, $T_2(\{u\}^{(0)},t)$, $T_3(\{u\}^{(0)},t)$ which is inserted – as the well-known – the mathematical model found in the previous step. Is obtained $\overline{u^{(1)}}$ and so on.

- Iterations stop when $\left\|\overline{u^{(i)}} - \overline{u^{(i+1)}}\right\| < \varepsilon$, ∀x, ∀t, (1)

ε is chosen according to the desired accuracy in the calculation.

- Then $\vec{u}(x,t) = \overline{u^{(i+1)}}(x,t)$
and with the solution $\overline{u^{(i+1)}}(x,t)$, we can built – as in [5] – the characteristic parameters of random dynamic response.

In this situation, the input can be random:
- a random external load – as in [5]
- a random excitation displacement type basis.

2. DEVELOPMENT SUBJECT

Mechanical models with finite number of degrees of freedom is an approximation of the real mechanical model (with mass distributed).

These models are built by focusing mass cinematic elements at certain points, either empirically or on the basis of equivalent conditions.

Mathematical models for these mechanical models are systems of ordinary differential equations, which – as we shall see below -is a great advantage for determining the static parameters of the dynamic response.

It is considered linear elastic vibration rod – bar Bernoulli – Euler – concentrated mass M discrete points of a mechanism R (RRR) - figure 1 – with the basic random excitation known.

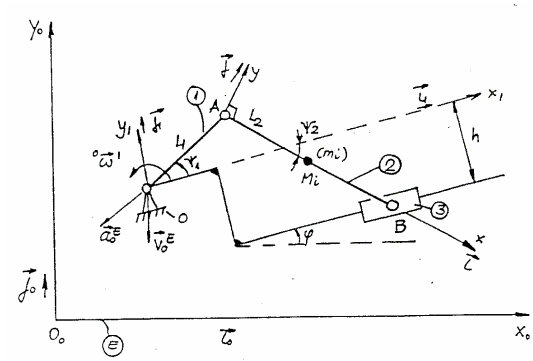

Figure 1. The cinematic bar element Bernoulli-Euler with the basic random excitation known

It is considered that the angular velocity $\overset{0\to 1}{\omega}$ of the crank 1 to base 0 of mechanism is constant:

$$\overset{0\to 1}{\omega} = \omega_{10}\vec{k}_1 \qquad (2)$$

It is considered that the mechanism has a planar random motion with kinematic parameters $\overset{\to E}{v_0}$; $\overset{\to E}{a_0}$; $\overset{E\to 0}{\omega}$; shuffle data.

We calculate the next three formulas (3):

$$\vec{v}^{\,E}_{0} = v_{0X}(t)\,\vec{i_0} + v_{0Y}(t)\,\vec{j_0};$$
$$\vec{a}^{\,E}_{0} = a_{0X}(t)\,\vec{i_0} + a_{0Y}(t)\,\vec{j_0};$$
$$\vec{\omega}^{\,E\,0}_{0} = \omega_0(t)\,\vec{k_0}; \quad \vec{\varepsilon}^{\,E\,0}_{0} = \varepsilon_0(t)\,\vec{k_0} \qquad (3)$$

It is considered that rototranslation instant flat rod 2 $\vec{v}^{\,E}_{A}$; $\vec{a}^{\,E}_{A}$; $\vec{\omega}^{\,E\,2}$;

We calculate the next three formulas (4):

$$\vec{v}^{\,E}_{A} = v_1(t)\,\vec{i} + v_2(t)\,\vec{j};$$
$$\vec{a}^{\,E}_{A} = a_1(t)\,\vec{i} + a_2(t)\,\vec{j};$$
$$\vec{\omega}^{\,E\,2} = \omega(t)\,\vec{k}; \quad \vec{\varepsilon}^{\,E\,2} = \varepsilon(t)\,\vec{k}. \qquad (4)$$

It is obtained elementary (5), (6), (7), (8):

$$a_1(t) = a_{0X}(t)*\cos(\Psi_2-\varphi) - a_{0Y}(t)*\sin(\Psi_2-\varphi) - L_1(\omega_{10}+\omega_0)^2*\cos(\omega_{10}t+\Psi_2); \qquad (5)$$

$$a_2(t) = a_{0X}(t)*\sin(\Psi_2-\varphi) - a_{0Y}(t)*\cos(\Psi_2-\varphi) - L_1(\omega_{10}+\omega_0)^2*\sin(\omega_{10}t+\Psi_2); \qquad (6)$$

$$\omega(t) = \omega_0(t) - \lambda\omega_{10}\left(1+\frac{\mu^2}{2}\right)*\cos(\omega_{10}t) \qquad (7)$$

$$\varepsilon(t) = \varepsilon_0(t) + \lambda\omega_{10}^2\left(1+\frac{\mu^2}{2}\right)*\sin(\omega_{10}t) \qquad (8)$$

We use the following notations:

$\lambda = \frac{L_1}{L_2}$;

$\mu = \frac{h}{L_2}$;

$\Psi_2 = \arcsin[\lambda\sin(\omega_{10}t) + \mu]$;

$\varphi(t) = \int_0^t \omega_0(u)$

We have neglected terms in λ^k, $k \geq 2$, λ is assumed small enough.

It focuses mass M of the element 2 in "n" concentrated masses m_i, $\sum_{i=1}^{n} M$ placed in the points M_i, $i=\overline{1,n}$.

A sufficiently accurate solution is obtained for the points M $m_i = \frac{M}{n}$, dividing the segment AB in the (n+1) equal parts.

We can write now (9):

$$\vec{a}^{\,E}_{M_i} = \vec{a}^{\,E}_{A} + \vec{\varepsilon}^{\,E\,2} \times \vec{r_j} + \vec{\omega}^{\,E\,2} \times \left(\vec{\omega}^{\,E\,2} \times \vec{r_j}\right) + \frac{\partial^2 \vec{u_i}}{\partial t^2} + 2\,\vec{\omega}^{\,E\,2} \times \frac{\partial \vec{u_j}}{\partial t} + \vec{\varepsilon}^{\,E\,2} \times \vec{u} + \vec{\omega}^{\,E\,2} \times \left(\vec{\omega}^{\,E\,2} \times \vec{u_i}\right) \qquad (9)$$

Linear elastic displacement of point M is (10):

$$\vec{r_j} = \overrightarrow{AM_i} = x_i\,\vec{i}; \qquad \vec{u_j} = u_i(t)\,\vec{i} + w_i(t)\,\vec{j}; \qquad (10)$$

Is noted $\vec{F}_{in_i} = X_{in_i}\,\vec{i} + Y_{in_i}(t)\,\vec{j}$; inertia force associated with the mass m.

We calculate the next two formulas (11), (12):

$$X_{in_i} = m_i\Big\{-a_{0X}\cos(\Psi_2-\varphi) + a_{0Y}\sin(\Psi_2-\varphi) + L_1(\omega_{10}+\omega_0)^2\cdot\cos(\omega_{10}t+\Psi_2) + \Big[\omega_0^2(t) + \frac{1}{4}\lambda^2\omega_{10}^2(2+\mu^2)^2\cos^2(\omega_{10}t)\Big]x_i - \frac{\partial^2 u_i}{\partial t^2} + 2\Big[\omega_0(t) - \lambda\omega_{10}\left(1+\frac{\mu^2}{2}\right)\cos(\omega_{10}t)\cdot\frac{\partial w_i}{\partial t}\Big] + \Big[\varepsilon_0(t) + \lambda\omega_{10}^2\left(1+\frac{\mu^2}{2}\right)\cdot\sin(\omega_{10}t)\Big]w_i + \Big[\omega_0^2 + \lambda^2\omega_{10}^2\cdot\frac{1}{4}\cdot(2+\mu^2)^2\cos^2(\omega_{10}t) - \omega_0(t)\cdot\lambda\omega_{10}(2+\mu^2)\cos(\omega_{10}t)\Big]w_i\Big\}; \qquad (11)$$

$$Y_{in_i} = m_i\Big\{-a_{0X}\sin(\Psi_2-\varphi) - a_{0Y}\cos(\Psi_2-\varphi) + L_1(\omega_{10}+\omega_0)^2\cdot\sin(\omega_{10}t+\Psi_2) - \Big[\varepsilon_0(t) + \lambda\omega_{10}^2\left(1+\frac{1}{2}\mu^2\right)\sin(\omega_{10}t)\Big]x_i - \frac{\partial^2 w_i}{\partial t^2} - 2\Big[\omega_0(t) - \lambda\omega_{10}\left(1+\frac{\mu^2}{2}\right)\cos(\omega_{10}t)\Big]\frac{\partial u_i}{\partial t} - \Big[\varepsilon_0(t) + \lambda\omega_{10}^2\left(1+\frac{\mu^2}{2}\right)\sin(\omega_{10}t)\Big]u_i + \Big[\omega_0^2 + \lambda^2\omega_{10}^2\cdot\frac{1}{4}\cdot(2+\mu^2)^2\cos^2(\omega_{10}t) - \omega_0(t)\cdot\lambda\omega_{10}(2+\mu^2)\cos(\omega_{10}t)\Big]w_i\Big\}; \qquad (12)$$

If it notes to α_{ij}, β_{ij}, coefficients that influence the transverse $w_i(t)$ and longitudinal $u_i(t)$ vibrations, we can write (13):

$$w_k = \sum_{i=1}^{n} \alpha_{ki}\left(Y_{in_i} + Y_i\right);$$
$$u_k = \sum_{i=1}^{n} \beta_{ki}\left(X_{in_i} + X_i\right); \qquad (13)$$

where $\vec{F}i = X_i\vec{i} + Y_i\vec{j}$ is the external force applied to the mass m.

Transport of an external force or an external couple in the points M is done by dynamic equivalence procedures known in mechanical dynamic systems material points' equivalent to a rigid solid.

We use the following notations:

$$\bar{\Psi}_i = -a_{0X}\sin(\Psi_2-\varphi) - a_{0Y}\cos(\Psi_2-\varphi) + L_1(\omega_{10}+\omega_0)^2 \cdot\sin(\omega_{10}t+\Psi_2) - \Big[\varepsilon_0(t) + \lambda\omega_{10}^2\left(1+\frac{1}{2}\mu^2\right)\sin(\omega_{10}t)\Big]x_i;$$

$$\Phi_i = -a_{0X}\cos(\Psi_2-\varphi) - a_{0Y}\sin(\Psi_2-\varphi) + L_1(\omega_{10}+\omega_0)^2\cdot\cos(\omega_{10}t+\Psi_2) + \Big[\omega_0^2(t) + \frac{1}{4}\lambda^2\omega_{10}^2(2+\mu^2)^2\cos^2(\omega_{10}t) - \omega_0(t) - \lambda\omega_{10}\left(1+\frac{\mu^2}{2}\right)\cos(\omega_{10}t)\Big]x_i$$

$$[A] = \begin{bmatrix} \alpha_{11}m_1; & \cdots; & \alpha_{1n}m_n \\ \vdots & & \vdots \\ \alpha_{n1}m_1; & \cdots; & \alpha_{nn}m_n \end{bmatrix} \qquad [B] = \begin{bmatrix} \alpha_{11}m_1\omega^2 - 1; & \cdots; & \alpha_{1n}m_n\omega^2 \\ \vdots & & \vdots \\ \alpha_{n1}m_1\omega^2; & \cdots; & \alpha_{nn}m_n\omega^2 - 1 \end{bmatrix}$$

$$[C] = \begin{bmatrix} \beta_{11}m_1; & \cdots; & \beta_{1n}m_n \\ \vdots & & \vdots \\ \beta_{n1}m_1 & \cdots; & \beta_{nn}m_n \end{bmatrix} \qquad [D] = \begin{bmatrix} \beta_{11}m_1\omega^2 - 1; & \cdots; & \beta_{1n}m_n\omega^2 \\ \vdots & & \vdots \\ \beta_{n1}m_1\omega^2; & \cdots; & \beta_{nn}m_n\omega^2 - 1 \end{bmatrix}$$

$$[E] = \begin{bmatrix} \beta_{11}; & \cdots; & \beta_{1n} \\ \vdots & & \vdots \\ \beta_{n1}; & \cdots; & \beta_{nn} \end{bmatrix} \qquad [F] = \begin{bmatrix} \alpha_{11}; & \cdots; & \alpha_{1n} \\ \vdots & & \vdots \\ \alpha_{n1}; & \cdots; & \alpha_{nn} \end{bmatrix}$$

$$\{w\} = \{w_1; w_2; \ldots; w_n;\}^T \qquad \{u\} = \{u_1; u_2; \ldots; u_n\}^T$$

$$\{\Psi\} = \{\bar{\Psi}_1; \bar{\Psi}_2; \ldots; \bar{\Psi}_n\}^T \qquad \{\Phi\} = \{\Phi_1; \Phi_2; \ldots; \Phi_n\}^T$$

$$\{X\} = \{X_1; X_2; \ldots; X_n\}^T \qquad \{Y\} = \{Y_1; Y_2; \ldots; Y_n\}^T;$$

From (13) we obtain the mathematical model system, consisting of (14) and (15):

$$[A]\{\ddot{w}\} + 2\omega \cdot [A]\{\dot{u}\} - [B]\{w\} + \varepsilon[A]\{u\} = [A]\{\Psi\} + [F]\{Y\}; \quad (14)$$

$$[C]\{\ddot{u}\} - 2\omega[C]\{\dot{w}\} - \varepsilon[C]\{w\} - [D]\{u\} = [C]\{\Phi\} + [E]\{X\}; \quad (15)$$

Solving mathematical model vary over time, consisting of systems (14) and (15) is a difficult problem for $n \geq 3$.

If (14) - (15) we neglect the influence of deformations, we obtained immediately the following linear system in hopes mathematics (16):

$$\begin{aligned} &[A]\{\ddot{m}_w\} + \{m_w\} = [F]\{m_Y\} \\ &[C]\{\ddot{m}_u\} + \{m_u\} = [F]\{m_X\} \end{aligned} \quad (16)$$

and that following the dispersion system (17):

$$\begin{aligned} &[A]\left\{\ddot{D}_w\right\} + \{D_w\} = [F]^2\{D_Y\} \\ &[C]\left\{\ddot{D}_u\right\} + \{D_u\} = [E]^2\{D_X\} \end{aligned} \quad (17)$$

where:

$$\{m_w\} = \{m_{w1}; w_{w2}; \ldots; w_{wn}\}^T;$$

$$\{m_u\} = \{m_{u1}; m_{u2}; \ldots; m_{un}\}^T$$

$$\{D_w\} = \{D_{w1}; D_{w2}; \ldots; D_{wn}\}^T;$$

$$\{D_u\} = \{D_{u1}; D_{u2}; \ldots; D_{un}\}^T$$

3. CONCLUSIONS

From (16), (17) are directly obtained statistical parameters dynamic response without prior resolution of the mathematical model, which is an obvious advantage.

If there are no simplifications previously announced, is going directly from the model (14) and (15) vary over time, with weightings given random function of time.

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