# MATHCAD APPLICATION TO DESIGN OF THE DISCRETE CONTACT LOOSE RIDING RINGS INCLUDED IN THE SUPPORTING SYSTEMS OF ROTARY DRUMS

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***Abstract:*** *The rotary drums are technological equipments used in manufacturing clinker cement materials, cooling cement clinker, cooling burnt pyrites exhausted from roasting furnace lines of sulfuric acid, cooling alumina discharged from calcination ovens, cooling lime exhausted from lime ovens, cooling slag from chemical fertilizer furnaces, etc. Rotary drums are also used in mixing of powder or granular materials, separation and enrichment of ore, coal, etc.., washing ores and other minerals, crystallization and extraction of sugar from sugar beets, etc.., drying various raw materials for cement clinker, clay, marl, furnace slag, coal, grain, etc.. This paper presents some contributions of application using MATHCAD 14© program into design and verification of loose riding rings mounted on support blocks included in the equipment's bearing group.*
***Keywords****: bearing groups, rotary drums, polar diagrams*

## 1. INTRODUCTION

The rotary drums support system is represented by continuous contact loose riding rings mounted on support blocks. They are important elements because in this way the loads are transmitted continuously from drum to the riding rings. This solution is used in case of very large (diameter 4 ... 5 m) and heavy aggregates (mass 35 ... 50 t) with large thermal expansion. These drums involve particular challenges in fabrication, installation and maintenance, considering that their average lifetime should be between 15 and 20 years with no major service [4]. Each bearing group consists of two cylindrical bearings placed symmetrically with respect to the vertical plane containing the axis of the drum (Fig. 1).

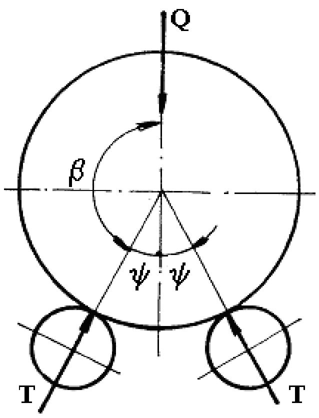


**Fig. 1 The riding rings supporting scheme**

The optimal position angle of the bearings with respect to the vertical diametral plane of the drums is $\psi = 30^0$. For higher values the roller's load increases and for lower ones the drum's stability decreases too much. The riding rings may be installed in different ways depending on the drum's size (and weight), temperature (thermal dilation) and other constructive aspects [1] … [5].

Thus, there are:
- loose riding rings mounted on support blocks with discontinuous contact; in this case, the load is transmitted from the drum to the riding rings by means of the support blocks. This mounting method is used for light and medium aggregates;
- loose riding rings mounted on support blocks with continuous contact; in this case the load is directly and continuously transmitted from the drum to the rings. This method is used in case of heavy large aggregates with significant thermal expansion;
- riding rings rigidly fixed to the drum, with discontinuous contact; in this case the load is transmitted directly through a number of equidistant points placed on the circumference (e.g. using removable bolted connections). This mounting method is used for light and medium aggregates;
- riding rings rigidly fixed to the drum, with continuous contact; in this case the load is transmitted directly through the welding seams on the contour. This method is used for relatively light and small aggregates when the thermal expansions are not significant.

In the riding rings appear stresses due to bending moments, axial forces on the ring section and contact forces in areas between ring and roller bearings. The size of these stresses depends on the method of mounting the ring on the drum.

In this paper are presented elements for calculating the bending moments and axial forces in the riding rings of rotating drums mounted on support blocks with discontinuous contact and the numerical results obtained in a particular case, using equations given by literature [1]…[7], and features of professional software package MATHCAD 14 [8] … [11].

## 2. BENDING MOMENTS EXPRESSIONS

Considering that the support blocks are equidistant (angle between two consecutive blocks is $\delta = 2 \cdot \pi / n$ - n number of blocks) and the load carried by these blocks is a cosine function and taking into account that the load depends on the blocks' position, the forces that they transmit to the riding ring are calculated for two extreme cases (Fig. 4.2):

**- *Case I* -** when the inferior support block is placed in the vertical diametral plane (Fig. 2). In this case the load transfer from the drum to the riding ring is done through an odd number of support blocks located below the horizontal plane containing the axis of the drum.

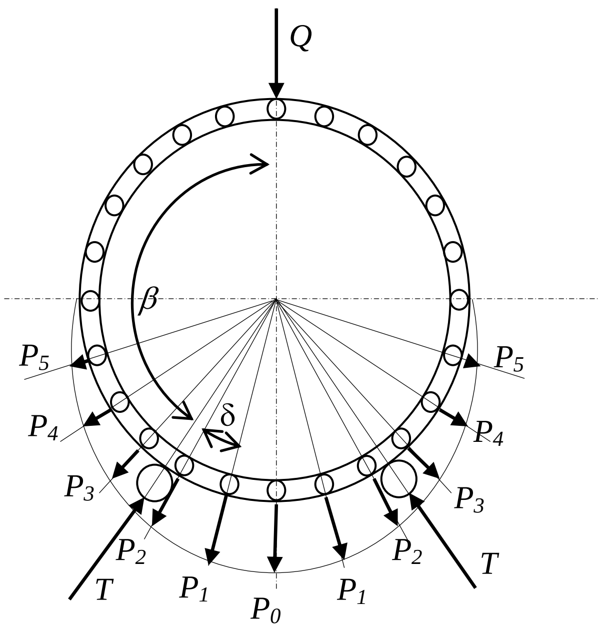


**Fig. 2 The loading scheme loose riding rings, when the inferior support block is placed in the vertical diametral plane (with 24 support blocks)**

The force is determined by the relationship [1]…[7]:

$$P_i = P_0 \cdot \cos(i \cdot \delta), \; i = 0, 1, 2, \ldots, n/2 \qquad (1)$$

where the force $P_0$ is determined by the relationship:

$$P_0 = \frac{Q}{1 + 2 \cdot \left[ \frac{k}{2} + \frac{\cos(k+1)\delta \cdot \sin k\delta}{2 \sin \delta} \right]} \qquad (2)$$

and $k$ is the integer part of the ratio *(n-2)/4*.

**- *Case II* -** when two consecutive support blocks are symmetrically placed with respect to the vertical plane (fig.3). In this case the load transfer from the drum to the riding ring is done through an even number of equidistant blocks placed under the horizontal plane containing the axis of the drum.

The force is determined by the relationship:

$$P_i = P_0 \cdot \cos[(i - 0.5) \cdot \delta], \quad i = 1, 2, \ldots, n/4 \qquad (3)$$

where the force $P_0$ is determined by the relationship [1,2,3] :

$$P_0 = \frac{Q}{2 \cdot \left[ \frac{k}{2} + \frac{\cos k\delta \cdot \sin k\delta}{2 \sin \delta} \right]} \qquad (4)$$

and $k$ is the integer part of the ratio *n/4*

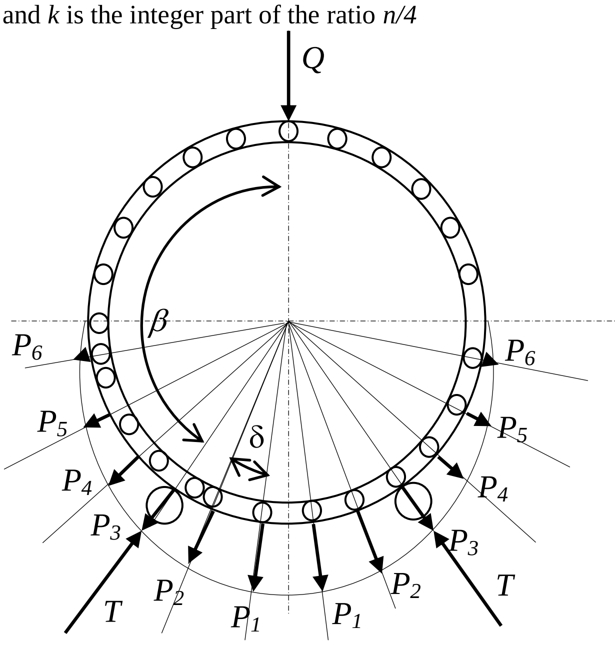


**Fig. 3 The loading scheme loose riding rings (two consecutive support blocks are placed symmetrically with respect to the vertical diametral plane)**

The riding rings are considered as small curvature circular beams, loaded with a number of pairs of radial loads which vary with their position. Bending moments are determined by superimposing the effects produced by each pair of radial loads.

Bending moments produced by a single pair $P_i$ of equal and symmetric radial loads, whose position is defined by the angle $\alpha_i$ (Fig. 4), is determined using the following relations:

- for $0 \le \varphi \le \alpha_i$ : $M = M_{0i} + N_{0i} R (1 - \cos \varphi)$ (5)

-for $\alpha_i \le \varphi \le \beta$ :

$$M = M_{0i} + N_{0i} R (1 - \cos \varphi) + P_i R \sin(\varphi - \alpha_i) \qquad (6)$$

-for $\beta \le \varphi \le \pi$ :

$$M = M_{0i} + N_{0i} R (1 - \cos \varphi) + P_i R \sin(\varphi - \alpha_i) - T_i R \sin(\varphi - \beta) \qquad (7)$$

where:

$\varphi$ is the center angle of the current section where the bending moment is calculated,

$R$ – radius of the ring's average circumference

$T_i$ – the roller bearings reaction is determined from the condition of equilibrium of forces on a vertical direction:

$$T_i = -P_i \cdot \frac{\cos\alpha_i}{\cos\beta} = P_i \cdot \frac{\cos\alpha_i}{\cos\psi} \qquad (8)$$

Since the ring's curvature is small, bending moments and axial sectional forces in middle top section are determined using CASTIGLIANO's theorem and they have expressions [1]…[7]:

$$M_{0i} = -\frac{P_i R}{\pi}\left[1 - \frac{\cos\alpha_i}{\cos\beta} - (\pi - \alpha_i)\sin\alpha_i + (\pi - \beta)\cos\,\alpha_i\; tg\,\beta\right]$$

$$N_{0i} = -\frac{P_i}{\pi}\left[(\pi - \alpha_i)\sin\alpha_i - (\pi - \beta)\cos\alpha_i\, tg\beta\right] \qquad (9)$$

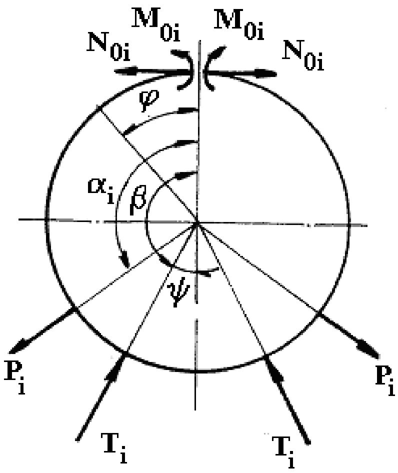


**Fig. 4. The design scheme of a loose riding ring mounted on support blocks, loaded with a pair of equal and symmetrical radial forces $P_i$**

For the case of a ring simultaneously loaded with several pairs of radial forces, the superposition principle will be applied. Thus the polar diagram bending moments diagram will be obtained. For the fatigue design of riding ring, the extreme values of positive and negative moments $M_{max}$ and $M_{min}$ are very important. One takes into account the highest value of maximum moment and lowest value of minimum moment, obtained for extreme loading case I and II.

## 3. AXIAL FORCES EXPRESSIONS

To determine the axial forces of the ring sections, one will proceed in a similar way as in the case of bending moments. In this case the forces depend as well on the mounting of the riding ring on the drum. For the loose riding ring mounted on support blocks, the relations for determining the normal forces *N* are:

-for $0 \le \varphi \le \alpha_i$: $N = -N_0 + \cos\varphi$ (10)

-for $\alpha_i \le \varphi \le \beta$: $N = -N_0 \cos\varphi + P_i \sin(\varphi - \alpha_i)$ (11)

-for $\beta \le \varphi \le \pi$:

$$N = -N_0 \cos\varphi + P_i \sin(\varphi - \alpha_i) - T_i \sin(\varphi - \beta) \qquad (12)$$

where:

$\varphi$ is the center angle of the current section where the normal force is calculated,

$T_i$ - the roller bearings reaction is determined from the condition of equilibrium of forces on a vertical direction with relation (8).

For case of simultaneous loading with several pairs of radial forces, the **superposition principle** will be applied. Thus the polar diagram of axial forces will result.

Since the normal forces are much lower than the bending moments, they are neglected in the fatigue design of these components.

## 4. MATHCAD APPLICATION OF POLAR DIAGRAMS

One will determine the bending moments and axial forces diagrams for the riding ring belonging to the supporting system of a rotary dryer. Numerical values of parameters are known:

- Drum diameter *D = 1.6 m;*
- Load in the support: *Q = 75,000 N;*
- Number of support blocks: *n = 24;*
- Riding ring geometry $D_{B=}$ *1990 mm;* $D_b$ *= 1760 mm;* $D_m$ *= 1875 mm; b = 135 mm; h = 115 mm.* (fig.5)

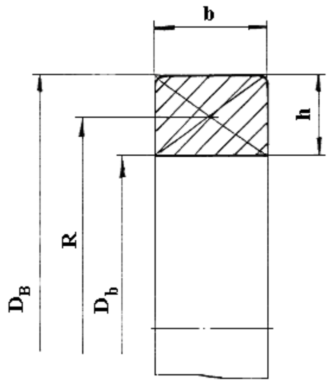


**Fig. 5. Scheme of riding ring geometry**

- The riding ring's average radius is: $R = \frac{D_m}{2} = 0{,}9375\ m$
- The center angle between two consecutive support blocks: $\delta = \frac{360^0}{24} = 15^0$.
- The angle between the two bearing rollers with respect to the vertical diametral axis of the drum is (Figure 4): $\psi = 30^0$ $(\beta = 150^0)$
- The bearing roller reaction on the riding ring is:

$$T = \frac{Q}{2\cos\Psi} = 43.300\ N$$

### 4.1. Case I - when the inferior support block is placed in the vertical diametral plane

The force $P_0$ loading the support blocks is determined by equation (2):

$$P_0 = \frac{Q}{1 + 2\left[\frac{k}{2} + \frac{\cos(k+1)\delta\,\sin k\delta}{2\sin\delta}\right]} = 12.500\ N$$

where $k$ is the number of support blocks pairs on the inferior half of the circumference: $k = \frac{n-2}{4} = 5$

The forces loading the support blocks will be determined by equation (1):

$$P_i = P_0 \cos(i\delta) \qquad i = 0..k\,.$$

The bending moments $M_{0i}$ and axial forces $N_{0i}$ in the upper middle section will be determined using the relations (9) and (10), where $\alpha_i = \pi - i\delta$ , $i = 0 \ldots k$.

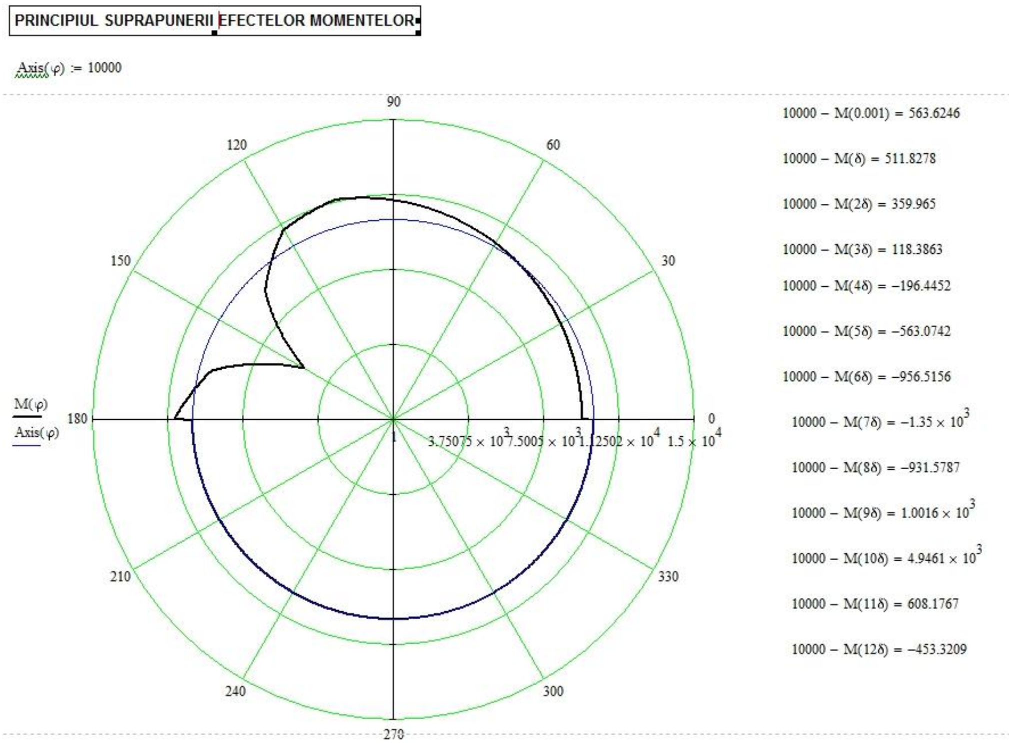


**Fig. 6. Bending moments diagram obtained for case I**

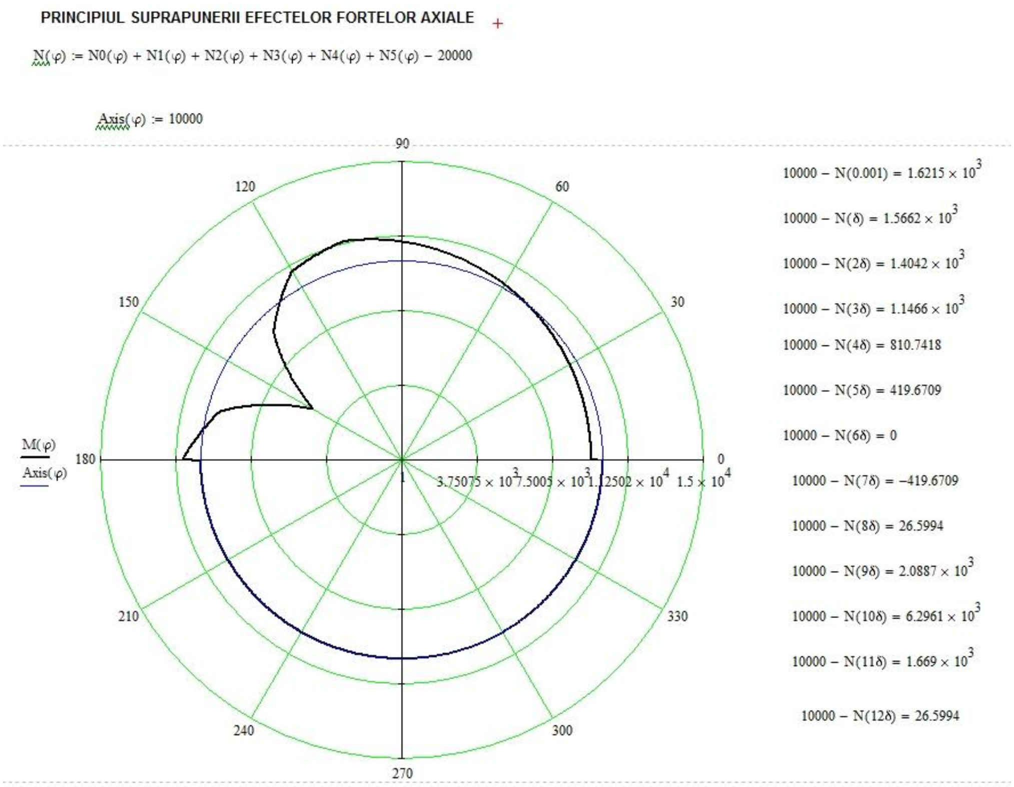


**Fig. 7. Axial forces diagram obtained for case I**

***a. Diagram of bending moments.*** The variation of bending moments on the riding ring is separately determined for each case of loading with forces pairs $P_1$, $P_2$, ... $P_6$ using equations (5) (6) and (7).
By superposition of effects we obtain the final diagram of bending moments for all six forces $P_1$, $P_2$, ... $P_6$ . Figure 6 shows the variation diagram of bending moments obtained by superimposing the effects for the six forces $P_1$, $P_2$, ... $P_6$ using MATHCAD 14 © [8]. Figure 6 gives also the numerical values of moments in the sections corresponding to the support blocks (for angles $\delta$, $2\delta$, ... $12\delta$) and to the roller bearings.

***b. Diagram of axial forces.*** Variation of axial forces is separately determined for each case of loading forces pairs $P_1$, $P_2$, ... $P_6$ using equations (11) (12) and (13). By the superposition of effects we obtain the final diagram of axial forces for all six forces $P_1$, $P_2$, ... $P_6$.
Figure 7 shows the variation diagram of axial forces obtained by superimposing the effects for the six forces $P_1$, $P_2$, ... $P_6$ using MATHCAD 14 program [8]. Figure 7 gives also the numerical values of axial froces in the sections corresponding to the support blocks (for angles $\delta$, $2\delta$, ... $12\delta$) and to the roller bearings ($\beta = 10\delta$).

### 4.2. Case II - when the two inferior support blocks are placed symmetrically with respect to the vertical diametral plane

The force $P_0$ loading the support blocks is determined by equation (4):

$$P_0 = \frac{Q}{2\left[\dfrac{k}{2} + \dfrac{\cos(k+1)\delta \sin k\delta}{2\sin\delta}\right]} = 12.500\ N$$

where $k$ is the number of support blocks pairs on the inferior half of the circumference: $k = \dfrac{n}{4} = 6$

The forces loading the support blocks will be determined by equation (1):

$$P_i = P_0 \cos(i + 0.5)\delta$$

The bending moments $M_{0i}$ and axial forces $N_{0i}$ in the upper middle section will be determined using the relations (9) and (10):

$$M_{0i} = -\frac{PR}{\pi}\left[1 - \frac{\cos\alpha_i}{\cos\beta} - (\rho - \alpha_i)\sin\alpha_i + (\pi - \beta)\cos\alpha_i\ tg\,\beta\right]$$

$$N_{0i} = -\frac{P}{\pi}\left[(\pi - \alpha_i)\sin\alpha_i - (\pi - \beta)\cos\alpha_i\ tg\beta\right],\quad \alpha_i = \pi - \left(i + \frac{1}{2}\right)\delta$$

***a. Diagram of bending moments.*** The variation of bending moments on the riding ring is separately determined for each case of loading with forces pairs $P_1$, $P_2$, ... $P_6$ using equations (5) (6) and (7). By superposition of effects we obtain the final diagram of bending moments for all six forces $P_1$, $P_2$, ... $P_6$
Figure 8 shows the variation diagram of bending moments obtained by superimposing the effects for the six forces $P_1$, $P_2$, ... $P_6$ using MATHCAD 14 [8]. Figure 8 gives also the numerical values of moments in the sections corresponding to the support blocks (for angles $\delta/2$, $\delta+\delta/2$... $11\delta+\delta/2$) and to the roller bearings .

***b. Diagram of axial forces.*** The variation of axial forces on the riding ring is separately determined for each case of loading with forces pairs $P_1$, $P_2$, ... $P_6$ using equations (11) (12) and (13). By superposition of effects we obtain the final diagram of axial forces for all six forces $P_1$, $P_2$, ... $P_6$
Figure 9 shows the variation diagram of axial forces obtained by superimposing the effects for the six forces $P_1$, $P_2$, ... $P_6$ using MATHCAD 14 program [8]. Figure 9 gives also the numerical values of forces in the sections corresponding to the support blocks (for angles $\delta/2$, $\delta+\delta/2$, ... $11\delta+\delta/2$) and to the roller bearings ($\beta = 10\delta$).

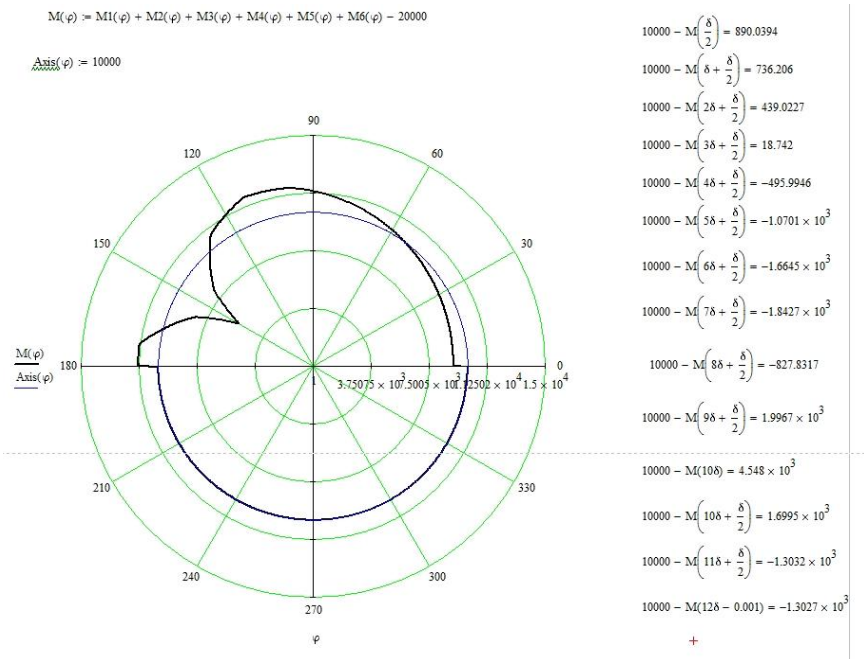


**Fig. 8. Bending moments diagram obtained for case II**

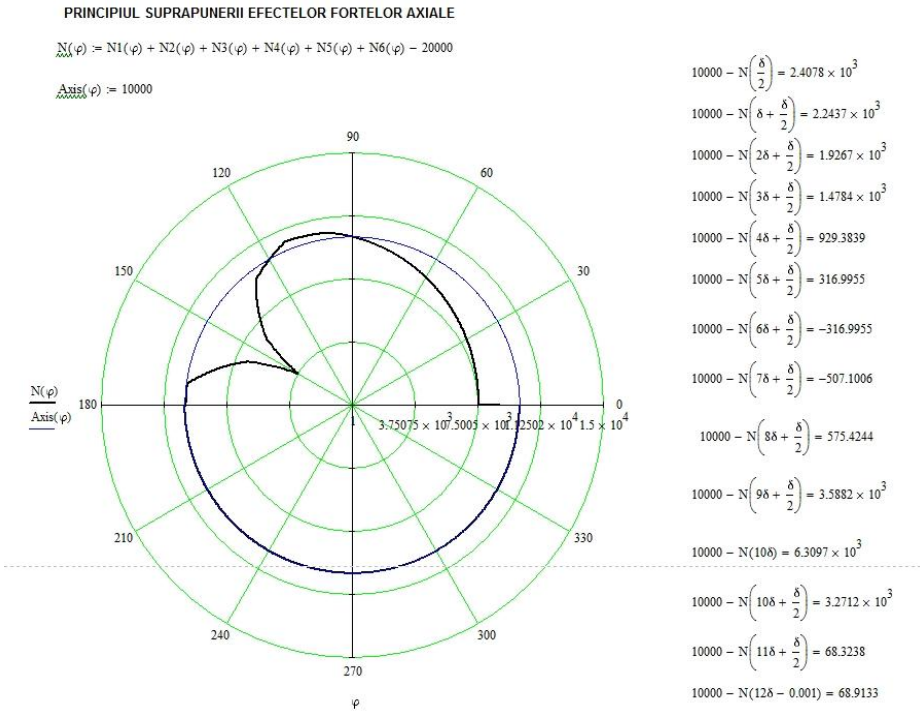


**Fig. 9 Axial forces diagram obtained for case II**

## 5. CONCLUSIONS

- After analyzing the bending moments diagrams in Figures 6 and 8 shows that the maximum (positive) moment is obtained in loading case I in the section corresponding to the roller bearings: $\varphi = 10\delta$; the maximum value of bending moment is: $M_{max} = 4946{,}1\ Nm$. The minimum (negative) is reached in loading case II in the section corresponding the angle $\varphi = 7\delta + \delta/2$; the value is $M_{min} = 1842{,}7\ Nm$
- The two extreme values of moments are necessary to determine the fatigue loads in alternating bending loading cycles of the riding ring.
- The maximum and minimum axial forces occur in the same sections and load cases, but they are very small and can be neglected in the fatigue design.
- Once completed this MATHCAD 14 © program, can be easily used for different loading cases simply by changing the input numerical data. The final polar diagrams will obtained instantly; so this method is well suited for optimization purposes of engineering design.

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